PHY 341 HW Ch.2d

q2-12
Find the energies and wave functions of the half SHO potential

$$V(x) = \begin{cases} \infty, & \text{if } x \le 0, \\ \frac{1}{2}m\omega^2 x^2, & \text{if } x > 0. \end{cases}$$

Hint: Do it the eeezeee way by picking from the full SHO solutions those that happen to also satisfy the half-space boundary conditions, $\psi(0) = \psi(\infty) = 0$.

q2-13
The initial wave packet is given by a Gaussian $\Psi(x, 0) = A\exp(-ax^2)$. (a) Find the normalization constant A. (b) Find the momentum wave function $\phi(k)$. (c) Roughly estimate (i.e., no need to calculate exactly) Δx and $\Delta p = \hbar\Delta k$, and show $\Delta x \Delta p \ge \hbar/2$.

q2-14
The wave function of a particle is given by $\psi(x) = A\exp(-\alpha|x|)$. Let $\phi(p)$ be the wave function in momentum space (no need to calculate), and Δp a measure of the "spread" of $\phi(p)$. As α is doubled, Δp will approximately
(A) double because $\Delta p \propto \alpha$
(B) stay the same because α is irrelevant to Δp
(C) be halved because $\Delta p \propto 1/\alpha$
(D) increase by four-fold because $\Delta p \propto \alpha^2$
(E) decrease by 1/4 because $\Delta p \propto 1/\alpha^2$

q2-15 The momentum wave function for a square wave is

$$\phi(k) = \sqrt{\frac{a}{\pi}}\,\frac{\sin(ka)}{ka},$$

with $p = \hbar k$ as usual. Show that $\phi(k)$ is normalized in k-space, $\int_{-\infty}^{\infty} |\phi(k)|^2\, dk = 1$. Use computer algebra or look up from a table of integrals.

q2-16 Let the energy of a particle be $E = \sqrt{E_0^2 + p^2c^2}$. Using $p = \hbar k$, find the "dispersion" relation $\omega(k)$, and obtain the group and phase velocities. What happens when $E_0 = 0$? Explain.

q2-17 Evaluate (a) $\int_{-\infty}^{0}(x^2+4)\delta(x-2)dx$ and (b) $\frac{d^2|x|}{dx^2}$ (express it in terms of a well-known function).

Useful integral (or just use sympy):
Let $I = \int_{-\infty}^{\infty} e^{-ax^2+bx}\, dx$ $(a > 0)$. Make a variable substitution, $x = y + \frac{b}{2a}$, then

$$I = e^{b^2/4a}\int_{-\infty}^{\infty} e^{-ay^2}\, dy = \sqrt{\frac{\pi}{a}}\, e^{b^2/4a}$$

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