PHY 341 HW Ch.3a

Do problems 3.4; plus the following (* = optional bonus):

q3-1
For each of the following wave functions, sketch it and determine whether it is a Hilbert space wave function ($a > 0$, $-\infty < x < \infty$):
(a) $A\exp(-ax)$; (b) $A\sin(kx)$; (c) $A\exp(-ax^2)$; (d) $A/\sqrt{-x}$; (e) $A\exp(-a|x|^2)/x^{1/4}$.

q3-2
Which of the following operators are Hermitian? Briefly explain.
$d/dx,\ d^2/dx^2,\ ix,\ a_+,\ a_-,\ a_+a_-,\ i\partial/\partial t$.

q3-3
Consider a particle in the box with ψ_n as the stationary states.
(a) Determine whether ψ_n are eigenfunctions of the Hamiltonian H, momentum $\hat{p}$, position $\hat{x}$, and momentum squared $\hat{p}^2$.
(b) Are H and $\hat{p}^2$ mutually compatible? What about $\hat{p}$ and $\hat{p}^2$? Briefly explain.

q3-4
Calculate the following inner product:
(a) $\langle a|b\rangle$ and $\langle b|a\rangle$, where $a = [1, 3, 2i, -2]^T$, $b = [-i, -1, i, 1]^T$.
(b) $\langle f|g\rangle$ where $f = e^{ikx-x}\sin 2x$, $g = e^{ikx-x}$, $0 \le x < \infty$, and k is real.

q3-5 Earlier we proved $[f(x), \hat{p}] = i\hbar f'$ in **q2-4**. Now prove this explicitly in momentum space. Hint: Let $\Phi = \Phi(p)$ be a test wave function in momentum space, and $\Phi^{(n)} \equiv \frac{\partial^n \Phi(p)}{\partial p^n}$. First show that $(p\Phi)^{(n)} = p\Phi^{(n)} + n\Phi^{(n-1)}$, then expand $f(x)$ as a Taylor series, such that $[f(x), p] = \sum_n c_n [x^n, p]$ with $c_n = d^n f/dx^n|_{x=0}/n!$. Noting $x = i\hbar\partial/\partial p$, calculate the commutator of the n-th term with p, namely $[x^n, p]\Phi$. Finally reverse the sum as the Taylor series of the desired result.